DEPARTMENT GEO ENERGY

MONTANUNIVERSITÄT LEOBEN

WELL INTEGRITY PLATFORM

Strength Measurements Infographic

Strength of materials, also called mechanics of materials, is a field in engineering that studies the stresses and strains that arise in solid objects like beams, columns and shafts, when loads are applied to them. This sheet covers the main topics in strength of materials, and presents the most important equations.

STRESS, PRESSURE AND HARDNESS

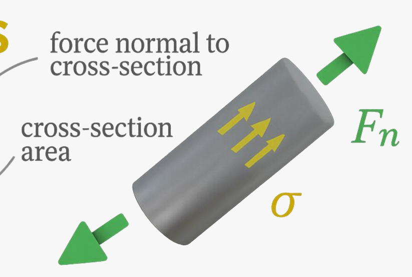
These three have all the same unit. Pressure and Stress even share similar mathematical equation and while both are described as force per unit area, their physical meaning, context and behavior are different. **Pressure** is the force applied on the surface of a material while **stress** is the internal force that develops inside the material as a response to external force. As for **Hardness**, it is a material's resistance to localized plastic deformation, such as indentation, scratching, or cutting and is measured by pressing a hard indenter into a surface.

STRESS AND STRAIN

Stress is a quantity that describes the distribution of internal forces that arise within a body in response to an applied load. It has units of force per area (N/m2 in SI units, also called Pascals and denoted as Pa). **Normal** stresses are stresses that act perpendicular to a surface and shear stresses are stresses that act parallel to a surface.

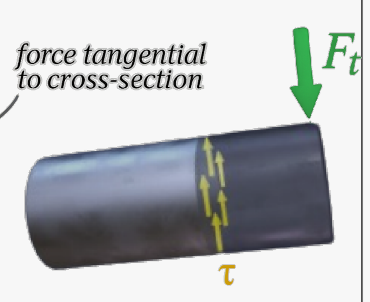
NORMAL STRESS

$$\sigma = \frac{F_n}{A}$$



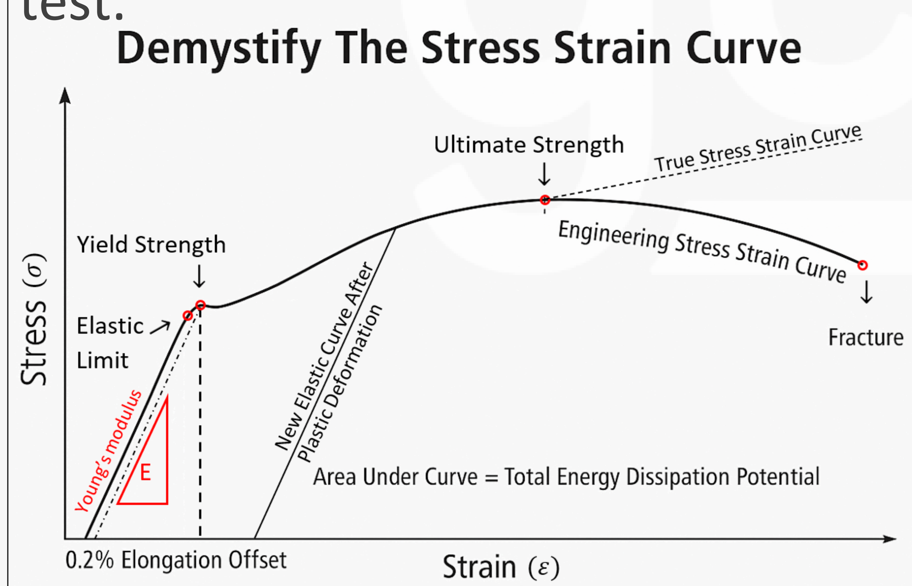
SHEAR STRESS

$$\tau = \frac{F_t}{A}$$



STRESS-STRAIN CURVE & HOOKE'S LAW

The relationship between stress and strain can be described using a stress-strain curve, obtained by performing a tensile test.



HOOKE'S LAW

$$\sigma = E\varepsilon$$

Hooke's law describes the relationship between stress and strain in the linear elastic region of the stress-strain curve.

Several material properties including Young's modulus, yield strength and ultimate strength can be determined from the stress-strain curve

The version of Hooke's law shown above only applies to uniaxial stress. For a general case of triaxial stress the generalized Hooke's law equations must be used.

GENERALIZED HOOKE'S LAW

$$\varepsilon_i = \frac{1}{E}[\sigma_i - \upsilon(\sigma_{i+1} + \sigma_{i+2})]$$

TRUE STRESS & TRUE STRAIN

In the typical stress-strain curve, stress is defined as the applied force divided by the original cross-sectional area of the test specimen. And strain is defined as the change in specimen length divided by the original length. These are just approximations of the stress and strain in the specimen, that are called engineering stress and engineering strain.

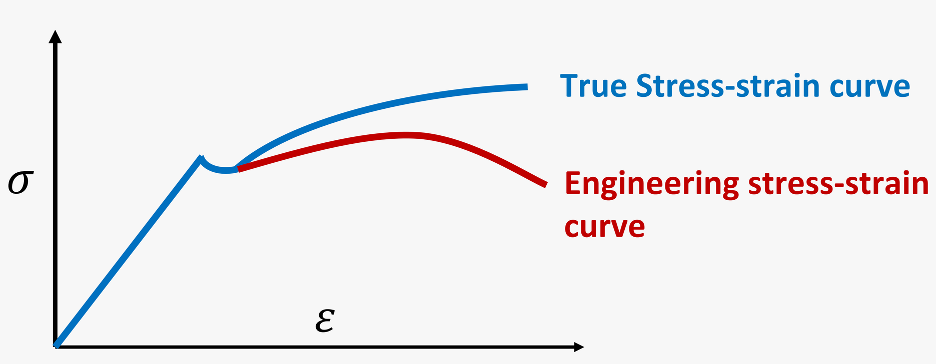
To determine the true stress and true strain values we need to consider the fact that the dimensions of the specimen change throughout the duration of the test.

TRUE STRAIN

$$\varepsilon_t = \ln(1 + \varepsilon)$$

TRUE STRESS

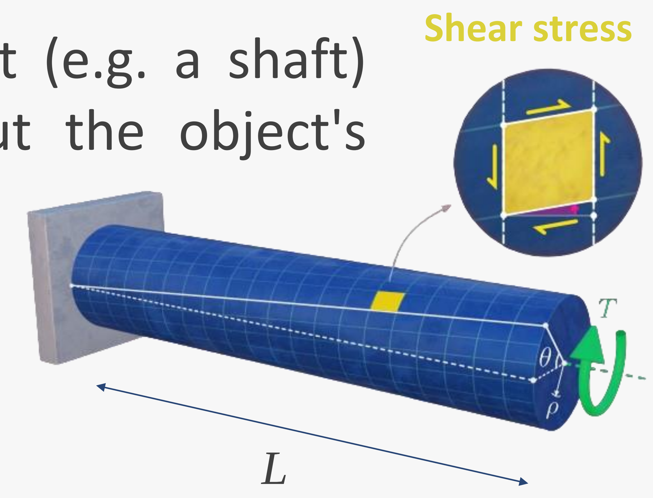
$$\sigma_t = \sigma(1 + \varepsilon)$$



TORSION

Torsion is the twisting of an object (e.g. a shaft) caused by a moment acting about the object's longitudinal axis (torque).

The applied torque T causes the end of the bar to rotate by an angle and generates shear stresses and strains in the bar.



L: bar length
T: applied torque
G: Shear modulus
J: Polar moment of inertia
ρ: Distance from the bar axis

ANGLE OF TWIST

$$\theta = \frac{TL}{GJ}$$

SHEAR STRAIN

$$\gamma = \frac{\rho\theta}{L}$$

SHEAR STRESS

$$\tau = \frac{T\rho}{J}$$

FAILURE THEORIES

Failure theories **for ductile materials** are used to predict failure of a material by comparing the triaxial stress state at a point with material properties that are easy to determine, like the yield or ultimate strengths obtained from a uniaxial tensile test.

The von Mises failure theory states that a ductile material has yielded when a calculated equivalent stress exceeds the yield strength of the material. According to this theory yielding occurs when the maximum distortion energy in the material is equal to the distortion energy at yielding in a uniaxial tensile test.

VON MISES EQUIVALENT STRESS

$$\sigma_{eq} = \sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]}$$

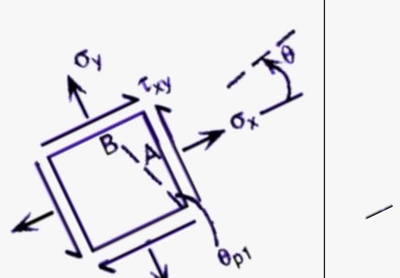
Failure theories **for brittle materials** are used to predict **cracking or fracture** by comparing the principal stresses at a point with the material's tensile and compressive strength (rather than yield strength). Unlike ductile materials, brittle materials like concrete fail suddenly, often when tensile stresses exceed their very low tensile strength.

Maximum principal stress theory (also known as Rankine theory) is a widely used failure criterion for brittle materials. This theory states that failure occurs when the maximum principal stress (σ_1) reaches the uniaxial tensile strength (ft) of the material. It does not consider shear stress or intermediate principal stress.

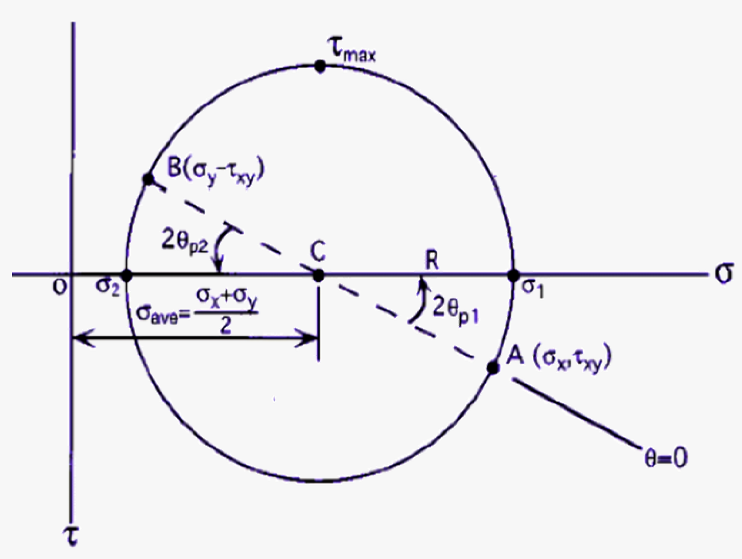
Mohr-Coulomb theory is more suitable for Concrete as it considers both normal and shear stresses. Concrete rarely yields — instead, it cracks under tensile stress. Therefore, predicting tensile cracking, shear failure, or crushing is essential in concrete design, not yielding like in metals.

STRESS TRANSFORMATION & MOHR'S CIRCLE

The stress state at a single point within a body will have components in the normal and shear directions. The stress element is a useful way of describing the stresses acting at a single point. The stress element on the right shows a 2D case (plane stress).



The magnitude of the normal and shear components depends on the angle of the plane being used to observe the stresses. These components can be determined graphically using Mohr's circle, or can be calculated using the stress transformation equations.



σ_1 and σ_2 are principal stresses where $\tau = 0$

STRESS TRANSFORMATION EQUATIONS

$$\sigma = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} cos2\phi + \tau_{xy} sin2\phi$$

$$\tau = -\frac{\sigma_x - \sigma_y}{2} sin2\phi + \tau_{xy} sin2\phi$$

https://geoenergy.engineering/
https://wip.geoenergy.engineering/

Name: Arash Nasiri
Arash.nasiri@unileoben.ac.at

CCS Booster & Bespoke Lab

Research Interests:
Carbon Capture and Storage (CCS)
Topological Investigation
Methane & Hydrogen Storage
Digitalization
Well Integrity

Strength Measurements Infographic

PROCEDURE

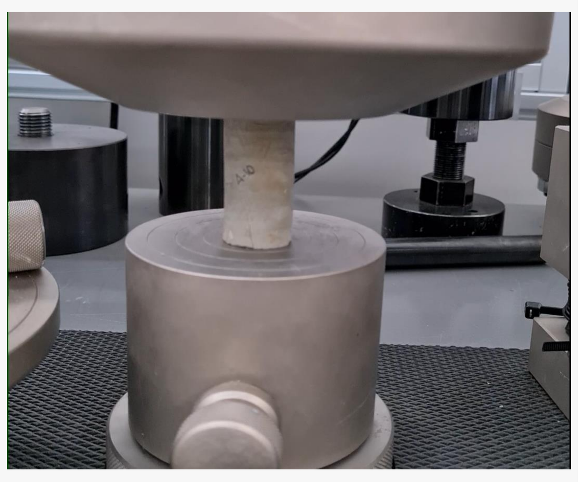
Main standards that are used for strength measurements are the following:

- **UCS Test**
  - **For Concrete:** ASTM C39, EN 12390-3, API 10B, ASTM C192, BS 1881-116, EN 12390-3
  - **For Rocks:** ASTM D7012, EN 17892-10, DGG(2004)
  - **Tor Metal:** ASTM E9, ISO 604, EN 6892-1
- **TCS Test**
  - **For Concrete:** No dedicated Standard --> Research protocols
  - **For Rocks:** ASTM D7012, EN 17892-9, BS 1377-8
  - **Tor Metal:** No dedicated Standard
- **Brazilian Test**
  - **For Concrete:** ASTM C496 (2011), EN 12390-6
  - **For Rock:** ASTM D3967 (2016)

Direct Tensile Test
For Metal: ASTM E8

Parameter	UCS Test (for Concrete)	Brazilian Test (for Concrete)
Standard Dimensions	• 50.8 mm cubes • 150 D×300 L mm cylinder	150 mm D × 50 mm t • $D = 2t$ (C496) • $D \sim t$ (EN 12390-6)
Curing Conditions	25 °C / 75 °C, up to 7 days	~28 days, 20 °C, >95% RH
Load Application	Uniaxial compression	Diametral compression
Load Rate (R_f) $[\frac{KN}{s}]$ **Stress Rate** (R_σ) $[\frac{MPa}{min}]$	• [1,08-1,31] KN/s (API 10B) • [2,4 - 4,8] MPa/min (EN 12390-3)	• [0,7-1,4] MPa/min (API 10B) • [12-24] MPa/min (BS 1881) • [2,4-3,6] MPa/min (EN 12390-6)
Displacement Rate (R_D) $[\frac{mm}{min}]$ According to EN 12390	1''x2'' sample: 6516,5 /E [MPa] 2''x5'' sample: 4072,86/E [MPa]	Depends on the angle of contact & hence the intersecting surface
Conversion:	$R_\sigma . \frac{L[mm]}{E[MPa]} \equiv R_D$ $R_f . \frac{60000 . L[mm]}{A[mm^2] . E[MPa]} \equiv R_D$	$R_\sigma . \frac{50 . A[m^2]}{3} \equiv R_f$

Uniaxial Compressive Strength Test (Cement Samples)



Depending on the control mode of the test, the piston will come down either with a constant speed or constant increase in the rate until the sample fails.

If the sample dimensions are not standard, in the sense that L<2d, owing to the **influence of end-surface friction** the true strength must be derived from the following equation:

**Note that for a valid test L/d should always be ≥ 1.5<*

** The external surfaces of the test specimens should be smooth and free of irregularities. The end surfaces should be flat and form right angles with the test specimen axis.*

UC Strength

$$\sigma = \frac{F}{A}$$

$$\sigma_2 = \frac{8 \times \sigma}{7 + 2\frac{d}{L}}$$

Triaxial Compressive Strength Test (Cement Samples)

TCS is a laboratory procedure used to evaluate the mechanical behavior of materials—particularly rocks, soils, and cementitious composites—under controlled multi-axial stress conditions that simulate in-situ confinement. In this test, a cylindrical sample is enclosed in a pressure cell (e.g., Hoek cell), surrounded by fluid to apply a lateral confining pressure, and then axially compressed using a universal testing machine. The procedure involves carefully preparing the sample, applying uniform confining pressure, and loading it at a controlled displacement rate until failure, while measuring axial and lateral strains using strain gauges or extensometers. Gained information like material's triaxial compressive strength, stress–strain response, elastic modulus, and Poisson's ratio under confinement are critical for geotechnical and petroleum engineering applications such as wellbore stability assessment, cement sheath integrity, reservoir compaction analysis, and underground storage design, where materials are subjected to both axial and lateral stresses

** The relationship between confining pressure(P_C) and the TCS(σ_T) for brittle materials are nonlinear. σ_T increases with increase in P_C.*

Considerations

✔ Fracture and crack propagation would differ with different loading rates. Such an increase results in a minimal increase in both fracture toughness and nominal strength(owing to decrease in fracture surface).

✔ Concrete strength under any boundary condition and loading configuration closely relates to the crack formation and propagation.

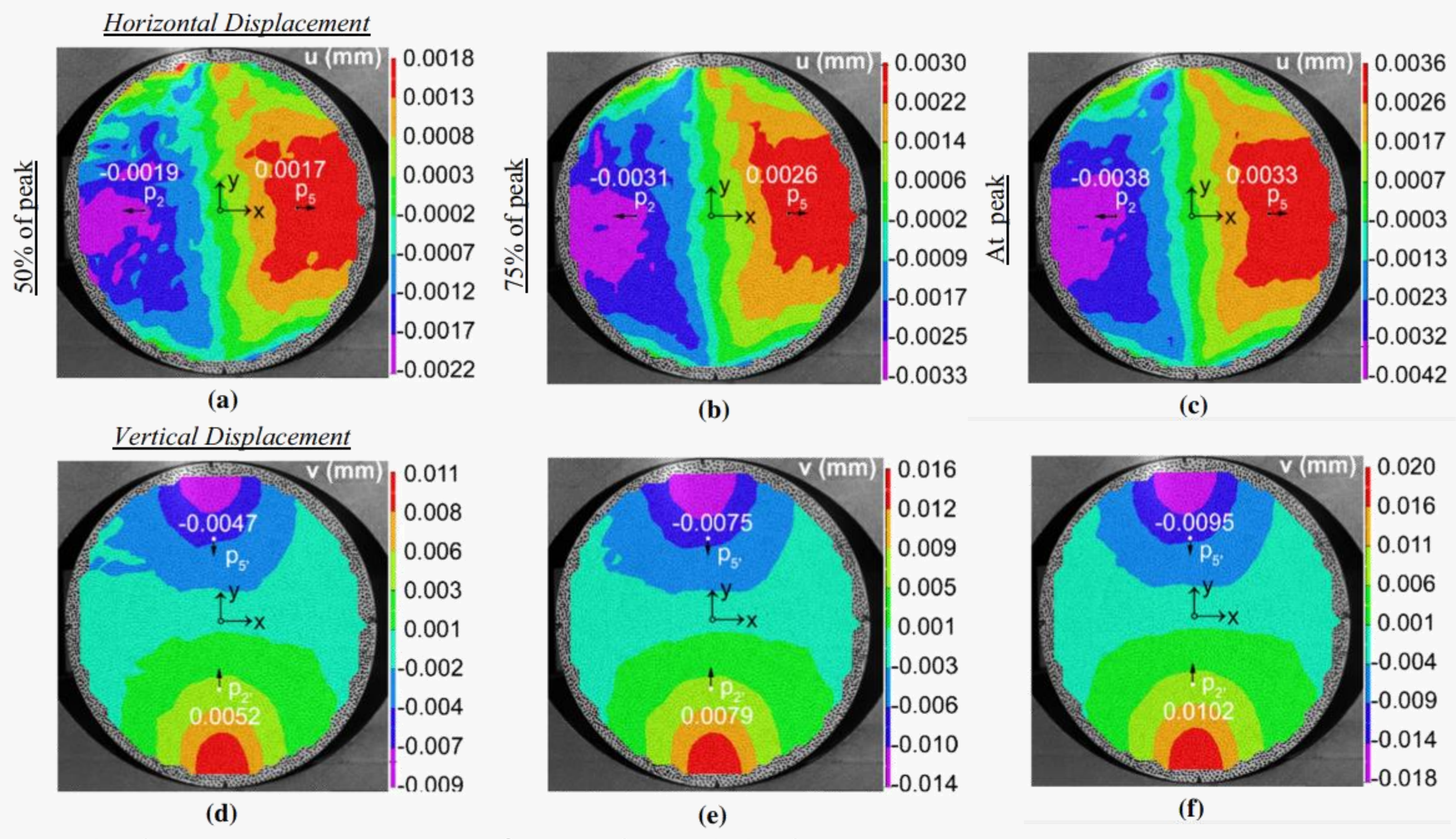
✔ German Geotechnical Society recommends a minimum of 5 minutes test (until failure) for both load and displacement control in rocks.

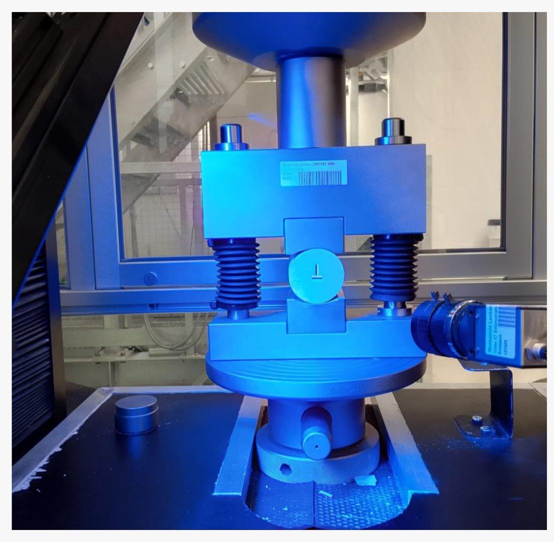
✔ To achieve the cylindrical shape, the samples must be processed as gently as possible by drilling, cutting, turning, grinding or other suitable processes, taking care to ensure that the heat input and the rinsing or cooling agents used change the properties of the material as little as possible.

Brazilian Test

The Brazilian Test (also called indirect tensile test or splitting tensile test) is used to determine the tensile strength of rock/cementitious samples, a property that is otherwise difficult to measure directly due to the material's brittle nature. It is crucial for understanding crack resistance, fatigue behavior, and durability of concrete in structural applications. Specimens must be moist-cured (usually for 28 days) before testing.

BT results have shown evidence that crack begins near to the loading points, which occurs because stress–strain field singularity that develops near to the loading points, and the stresses developed exceeds those developed at the center of the disk.

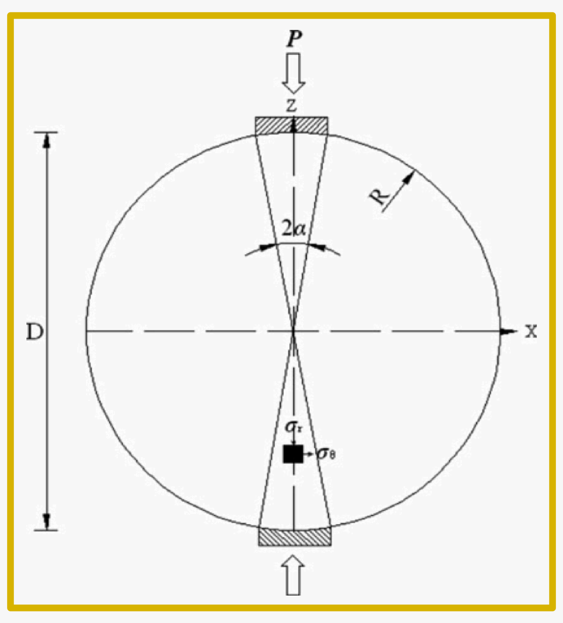




Displacement contours from the Digital Image Correlation(DIC) technique representing the horizontal and vertical strain in the sample(granite) under stress at three moments of 50% , 75% and 100% peak load. Note that horizontal displacement grow gradually from center to edge. And as expected largest vertical displacement appears on the point of contact.

Theoretical Background

Several researchers solved the stress equations of the circular element subjected to concentrated forces. Hondros (1959) took the next step and developed a complete stress solution for the case when the load is distributed over fin--ite arcs with diametral compression, valid for conditions of both plane stress (discs) and plane strain (cylinders).



His solution as well as the proceeding researches has the following general form.

The correction factor depends on the geometry and angle of contact. Different forms are suggested that can be found in related literature.

Splitting tensile Strength

$$\sigma_t = \frac{2P}{\pi D T} \times f_c$$

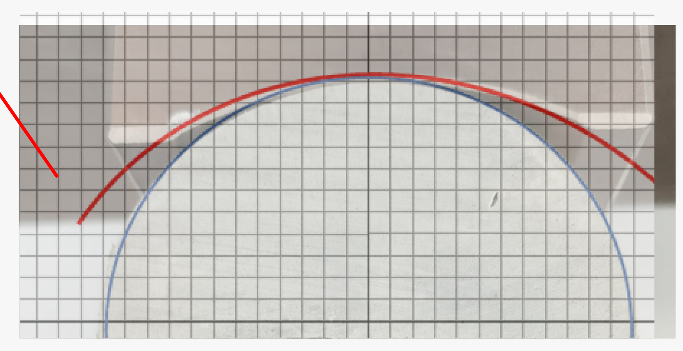
Regarding CDC laboratory, platens have Engineered curves with the following polynomial/Taylor fit:

P: Maximal Stress
σ_t: Tensile strength
D: Sample Diameter
fc: correction factor
T: Sample Thickness (equal to that of platen)

** The sample Diameter is 47 mm and it was used as the criterion for the grid scale. Then the exact fit was derived.*

$$y(x) = 11{,}9662 - 0{,}00709479x^2 - 2{,}0124 \times 10^{-6}x^4$$

Role of the Geometry of the Platens

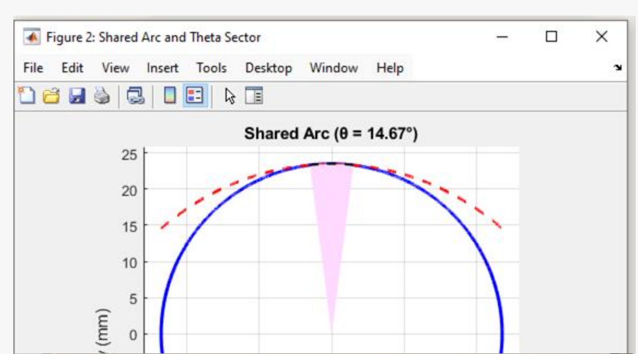


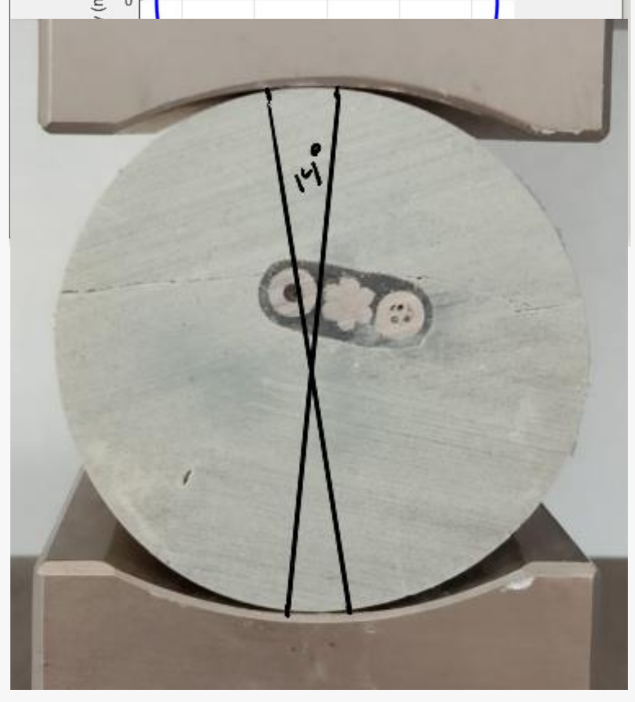
Depending on the geometry of the platens and the contact angle, the correction factor could be different

As regard our protocol, we follow the Hondros solution (Griffith criterion) meaning that:

Correction factor for the principal radial stress

$$f_c \,|_{\sigma_{tR}} = \frac{\sin 2\alpha}{\alpha} + 1$$





The contact angle, could be mathematically calculated by solving the y(x) for the outer premier of every sample with diameter D. As can be seen, for this case, sample has the diameter D=47 mm, and hence 2α is calculated to be 14,67° matching very well with the observation of 14 degrees.

https://geoenergy.engineering/
https://wip.geoenergy.engineering/

Name: Arash Nasiri
Arash.nasiri@unileoben.ac.at

Research Interests:
Carbon Capture and Storage (CCS)
Topological Investigation
Methane & Hydrogen Storage
Digitalization
Well Integrity